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AN EXPERIMENTAL STUDY OF PLANE BUBBLES RISING AT INCLINATION

C. C. MANERI and N. ZUBER

General Electric Company, Knolls Atomic Power Laboratory, Schenectady, New York, U.S.A.
Department of Mechanical Engineering, Georgia Institute of Technology, Atlanta, Georgia, U.S.A.

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Abstract—The characteristics of bubbles rising in two-dimensional tanks is investigated. It is found that the influence of fluid properties, while negligible for vertically rising bubbles, measurably affects the inclined bubble rise velocity. The measured increase in the rise velocity at inclination relative to the vertical value is explained qualitatively. In addition, a comparison is made between the wave analogy correlation of Maneri & Mendelson and vertical rise velocity data. The comparisons show that the correlation predicts the three-dimensional effects of tank spacing when the data are reduced on an equivalent bubble radius basis.

INTRODUCTION

The motion of bubbles in liquids has received considerable attention since Dumitrescu (1943) published his now classic paper on large bubbles rising vertically in tubes. The majority of investigators, however, have addressed themselves to determining the flow characteristics of bubbles rising vertically in finite and infinite media. Little attention has been given to bubbles propagating in systems at nonvertical orientations.

In recent years, the desire for such information has been motivated to a large degree by the nuclear reactor industry as a result of the proposed utilization of boiling reactors in seagoing vessels. Boiling is permitted to occur in order to improve the heat transfer characteristics of the coolant ducts; however, the presence of bubbles creates new problems which must be overcome in order to realize the heat transfer advantage. From a hydrodynamics standpoint, the pressure drop characteristics of the duct are altered to the extent that under certain conditions flow instabilities occur; whereas from a neutronics standpoint, the presence of bubbles influences core reactivity. Since ships experience pitch and roll (nonvertical conditions), it is necessary to consider the effects of these orientations on reactor operation.

The petroleum industry has long been concerned with the alternate flow of gas and liquid (slug flow) since crude oil is transported directly from the well through pipelines. Although the major effort in this area has been concerned with horizontal slug flow, it is evident that the topography of the land produces situations where the flow is inclined above the horizontal.

In view of the foregoing, it is clear that the motion of bubbles in liquids at nonvertical orientations can affect the operation of a variety of processes. To properly design the equipment in which these processes occur, it is necessary to determine the characteristics of inclined bubble motion. It is therefore the purpose of this paper to investigate the steady motion of bubbles rising in ducts at inclined orientations.

PREVIOUS WORK

The scientific literature contains the results of many investigations of the behavior of bubbles propagating in a stationary liquid. While the preponderance of previous workers has considered only vertically rising bubbles, the results are relevant to an investigation of the nonvertical case, since certain aspects of both systems are the same. In this sense, nonvertical bubble motion can be considered as an extension of the vertical case so that the results of these workers serve as the basis for physical interpretation of nonvertical systems. It is in this context that they deserve mention.

To begin with, it will be helpful to classify the studies that have been undertaken. This is most conveniently accomplished through the use of figure 1 which represents the entire range of regimes existing for three-dimensional bubbles. Figure 1 is essentially self-explanatory, but some remarks are necessary. This curve is typical of low viscosity liquids; regions 2 and 3 do not exist for highly viscous fluids. Another exception to this curve occurs when the test section has a small cross-sectional area. This situation results in the elimination of region 4.

In theory, a similar curve exists for two-dimensional or plane bubbles which are formed experimentally by introducing gas into a liquid contained between parallel plates or in a rectangular duct. In practice, however, a truly plane bubble cannot be obtained, since an actual bubble has curvature in the spacing direction. A further complication arises from the existence of a small liquid flow in a thin film on the faces of the plates. As a result, when the bubble dimensions are of the same order of magnitude as the spacing, the bubble can no longer be considered plane. Since the bubble sizes of regions 1 and 2 are exceedingly small, these regions cannot be obtained in systems of practical interest. In fact, the existence of region 3 also becomes questionable.

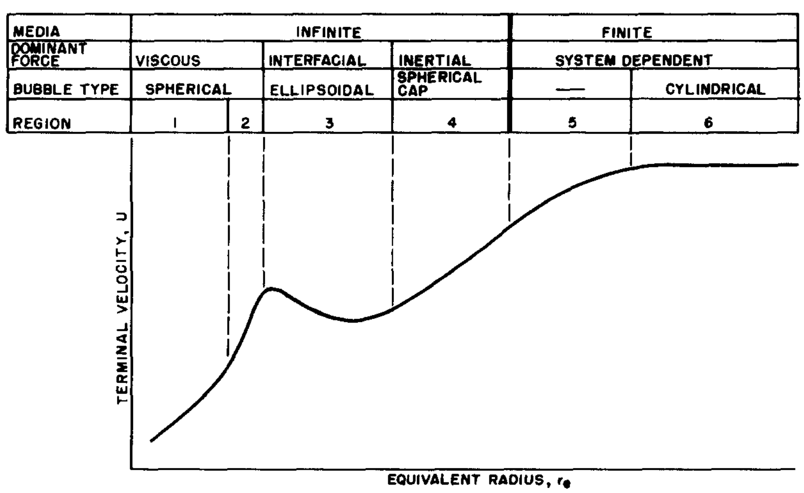


Figure 1. Bubble flow regimes.

Vertical bubble motion-infinite media

The subject of plane bubbles propagating in liquids of infinite extent has received attention only in recent years so that little experimental information exists. Collins (1965) obtained bubble velocity data on a "two-dimensional" tank having a $\frac{1}{4}$-inch (0.635 cm) spacing. He found his experimental values to be 9 per cent higher than the semi-theoretical result he derived for the rise velocity U_∞ using the approach of Davies & Taylor (1950)

$$U_\infty = \tfrac{1}{2}[ga]^{1/2}, \qquad [1]$$

where a is the frontal radius of curvature of the bubble and g is the acceleration of gravity. Collins (1965) attributed this discrepancy to additional flow along the faces of the plates.

Maneri & Mendelson (1968) were able to account for this effect in an extension of their wave analogy to two-dimensional geometries. They obtained the result

$$U_\infty = [g(2r_e + t)/\pi]^{1/2}, \qquad [2]$$

where t is the spacing and r_e is the two-dimensional equivalent radius defined in terms of the bubble volume V by the relation

$$r_e = [V/\pi t]^{1/2}. \qquad [3]$$

Grace & Harrison (1967) studied the influence of frontal shape on bubble velocity. They varied the frontal radius of curvature by inserting a vertical surface (rod) which the bubble could enclose. Their results show that the bubble velocity increases, reaching a limiting value as the frontal radius decreases. These authors were able to predict the limiting value from an expression derived by applying the approach of Davies & Taylor (1950) to an elliptical-cap bubble. This expression, however, cannot be utilized until, in addition to the bubble height and width, the major and minor axes of the ellipse corresponding to the elliptical-cap are known. Consequently, the real contribution of this work is the experimental observation.

Vertical bubble motion-finite media

The finite media regime as shown in figure 1 comprises two regions: a transition region (region 5) in which the presence of the containing boundary affects the terminal velocity, although it continues to increase with increasing volume, and a slug or long bubble region (region 6) in which the terminal velocity is independent of bubble volume. The investigation of the slug region will be discussed first because it is the natural limit of bubbles propagating in finite media.

Although the slug velocity is independent of bubble length, it is very dependent upon the characteristic dimension of the duct and the properties of the liquid. At least seven regimes (White & Beardmore 1962) have been defined in terms of various combinations of the Froude, Weber, and Reynolds numbers. However, this review will be primarily concerned with those regions where inertial forces predominate.

Griffith (1963) obtained slug velocity data in rectangular ducts having aspect ratios (spacing-to-width) ranging from 0.07 to 1.0. He found the large dimension to be the significant one, although the Froude number increased with increasing aspect ratio. For thin channels, the Froude number, based on the duct diameter, is close to, but greater than, the theoretical value of 0.23 ± 0.01. The data of Collins (1965) exhibit the same trend, although Collins did not recognize it. He therefore quoted the average of his data, 0.247, for the experimental Froude number.

Using the approach of Davies & Taylor, Collins (1965) derived an expression for two-dimensional bubbles, although he did not relate the frontal radius of curvature to the volume. His expression yields the correct theoretical velocity for the infinite media and slug limits, although the latter result is fortuitous, since the corresponding ratio of frontal radius-to-channel width is 0.478, whereas the experimental result is about 0.31.

Maneri & Mendelson (1968) extended the wave analogy to encompass finite systems. The main feature of the analogy is suitable interpretation of the wavelength and liquid height of wave theory in terms of characteristic dimensions of the bubble and duct, respectively. Their expressions for both three-dimensional and two-dimensional media are in good agreement with existing experimental data. The two-dimensional result is an improvement on Collins' analysis in that it accounts for three-dimensional inertial effects.

Inclined bubble motion

White & Beardmore (1962) were the first to recognize an inclination effect on the propagation of bubbles in tubes; however, their intent was not to investigate this effect but to point out the necessity for careful positioning of the test section.

The first comprehensive investigation of slugs in inclined tubes was performed by Runge & Wallis (1965) for a wide range of fluids. These authors noted nonsystematic behavior of their results with respect to Eötvös number and an inverse viscosity number, $(g^{1/2}D^{3/2}\rho/\mu)$, where D is the tube diameter and ρ and μ are the fluid density and viscosity. No theoretical analysis was attempted. Rather, the data are presented graphically with the above-mentioned groups as parameters.

Zukoski (1966) studied the behavior of slugs propagating through acetone and water in tubes at orientations ranging from vertical to horizontal. Using the vertical case as a basis, he concluded that viscous effects were negligible in these systems. As a result, he presented his data in terms of the Froude and Eötvös numbers. The results show that as the Eötvös number increases, the Froude number for a vertical tube rapidly approaches a limiting value, whereas at inclination it continues to increase. In conjunction with this, he noted that at inclination the nose radius changed with increasing Eötvös number from an appreciable to a very small fraction of the tube radius. Since previous theoretical treatments for the vertical case (Dumitrescu 1943; Davies & Taylor 1950) had shown the radius of curvature at the stagnation point to be critical in fixing the velocity, he concluded that as the Eötvös number approached zero the propagation rate for inclined tubes would also continue to change in response to the continued change in the nose-to-tube radius ratio.

This conclusion was shown to be incorrect by Benjamin (1968) who analyzed the motion of a long bubble in an ideal fluid in a horizontal tube. His theoretical value for the Froude number, 0.767, which is 2.2 per cent above Zukoski's highest experimental value, sets an

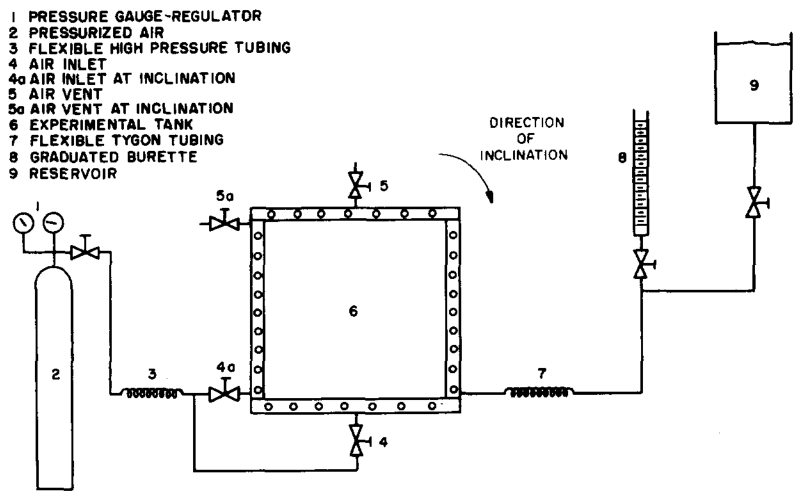


Figure 2. Schematic of experimental apparatus.

upper limit on Zukoski's data. Benjamin (1968) also analyzed the analogous two-dimensional case and found the theoretical Froude number to be 0.5. No comparison with experiment was made, however, since data were not available in the literature and he did not conduct his own experiments.

EXPERIMENTAL INVESTIGATION

A schematic of the experimental apparatus is given in figure 2. The 3-ft (0.914 m) length test tanks were constructed by separating two Plexiglas plates by aluminum spacers, and then clamping the assembly together along all four edges. In all, six tanks were constructed having nominal widths of 2.5, 6, or 34 inches (6.35, 15.24, or 86.36 cm) and nominal spacings of $\frac{3}{8}$ or $\frac{1}{2}$ in. (0.953 or 1.27 cm). Actual dimensions are given in table 1. Since the same spacers and clamps were used for the six tanks, the edges were sealed with nonadhesive Type Q Apiezon. The clamping process forced the Apiezon to flow and form a relatively uniform, airtight gasket.

Table 1. Actual cross-sectional dimensions for 34-in. (0.864 m) long tanks

Nominal dimensions (in.)	Nominal dimensions (cm.)	Width (in.)	Width (cm.)	Spacing (in.)	Spacing (cm.)
0.375 × 2.5	0.953 × 6.35	2.4671	6.2664	0.40634	1.03210
0.5 × 2.5	1.270 × 6.35	2.4606	6.2499	0.54420	1.38227
0.375 × 6.0	0.953 × 15.24	5.9981	15.2352	0.39868	1.01265
0.5 × 6.0	1.270 × 15.24	6.0346	15.3279	0.52616	1.33645
0.375 × 34.0	0.953 × 86.36	34.0	86.36	0.40087	1.01821
0.5 × 34.0	1.270 × 86.36	34.0	86.36	0.52679	1.33805

The tank support consisted of a 4-ft (1.22 m) square, $\frac{3}{4}$-in. (1.905 cm) thick plywood board
The tank support, consisted of a 4-ft (1.22 m) square, $\frac{3}{4}$-in. (1.905 cm) thick plywood board mounted on a 3-ft (0.914 m) high table in a manner which allowed the board to be rotated through 90 degrees from the vertical. Inclinations were measured with a protractor mounted in the upper left-hand corner of the board. Since data were to be obtained photographically utilizing a back-lighting technique, a rectangular window, 13 in. × 21 in. (33.02 × 53.34 cm), was made in the upper left-hand quadrant of the plywood board. The position of the window corresponded to the test section of the tanks which was taken to be the upper 12 in. (30.48 cm).

Two fluids were used in this investigation, deionized water and reagent grade methanol (methyl alcohol). Methanol was chosen primarily because its surface tension is less than one third of that of water, the largest difference encountered in systems of practical interest. Other considerations were its availability, relatively low cost, and the fact that it does not dissolve Plexiglas. Fluid properties were taken from the literature at 25°C, since the fluid temperature for all but three of the more than 150 series of runs remained between 20 and 30°C. Additional accuracy was unwarranted because of the weak variation of the fluid properties with temperature.

Individual bubbles were injected with a hand pump inflating needle soldered to a spring closing Hoke toggle valve. The valve was connected to a pressurized air bottle via $\frac{1}{4}$ in. (0.635 cm) flexible, high-pressure plastic tubing. A single bubble was injected into the tank by flicking the toggle valve, the size of the bubble depending upon the open-time of the valve. In this way, bubble volumes as large as 40 cm^3 were produced completely free of satellite bubbles. It was extremely difficult to prevent the formation of satellites for larger bubble volumes, although it is judged that their presence increased the volume measurements by at most 5 per cent. Once the slug limit was reached, the presence of satellites was inconsequential. Bubbles were injected at the center of the base when the tank was oriented vertically and at the lower left-hand corner when the tank was inclined.

The air injection method was used to produce bubble sizes up to and including the slug limit at all inclinations in the 2.5-in. (6.35 cm) tanks. In the 6-in. (15.24 cm) tanks, however, the incipient slug volume could not be obtained in a single injection at inclinations greater than 10 degrees. Attempts at introducing large volumes by increasing the open-time of the toggle valve resulted in the air streaming along the upper edge of the tank. As a result, slugs of "infinite" volume were produced by the tank-emptying method which was accomplished by filling the tank at the desired orientation and then quickly removing the bottom spacer.

Bubble volumes were measured in the smaller tanks by displacement of the test fluid. Since rising bubbles expand because of the decreasing pressure, the displaced fluid volume equalled the bubble volume at atmospheric pressure. The increase in bubble volume over the test section length of 12 in. (30.48 cm) was calculated to be 3 per cent assuming ideality of the air. This was considered a negligible effect since the bubble velocity varies approximately as the fourth root of the volume in the volume-dependent regime.

A different *modus operandi* for measuring the bubble volume in the 34-in. (0.864 m) tanks was necessitated by the fact that these tanks expanded slightly upon injection of the pressurized bubble volume. As a result, relatively large bubble volumes could be injected

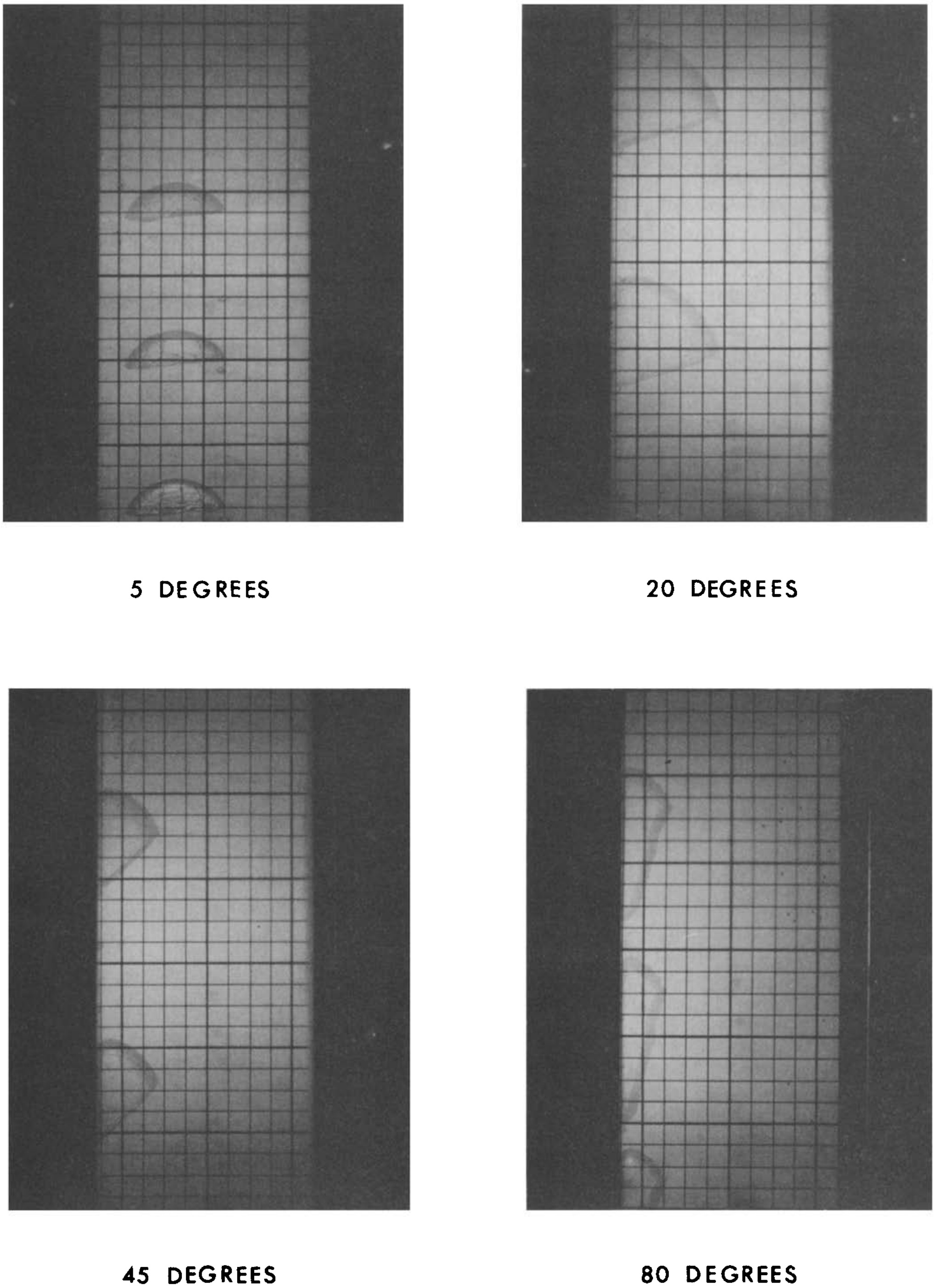


Figure 3. Typical examples of multiple exposures.

without fluid displacement. For example, a 2 per cent increase in average spacing, say from 0.5 in. (1.27 cm) to 0.51 in. (1.295 cm) would accommodate a bubble volume of 189 cm^3. Bubble volumes under inclined conditions were therefore calculated from the tank spacing, the angle of inclination, and the measured leg of the triangle formed by the air in the upper corner of the tank.

Volume measurements for vertically rising bubbles in the 34-in. (0.864 m) tanks were obtained by orienting the tank at 45 degrees after each run. This volume measuring technique was less accurate than the direct displacement method; however it is judged that the error is within 5 per cent for volumes greater than 20 cm^3 and may be as high as 15 per cent for smaller volumes.

The terminal velocity of a bubble was measured photographically on Type 52 Polaroid film, ASA No. 400, with a Speed Graphic camera having a 135 mm lens. A Type 1531A Strobotac stroboscope illuminated the test section from behind through the rectangular window. Covering the window on the back face of the test section was a sheet of white translucent Mylar on which was lined a $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. (0.635 cm $\times$ 0.635 cm) grid. In addition to providing a white background, the Mylar also served to diffuse the light from the Strobotac so that a uniformly illuminated background was obtained in the photographs. The procedure consisted of obtaining a multiple exposure of the bubble as it propagated along the test section, each exposure corresponding to a flash of the Strobotac. Typical examples of these photographs are shown in figure 3.

Velocity measurements were obtained vertically and at various inclinations up to and including 90 degrees. Only tank-emptying experiments were performed at 90 degrees (for the finite media tanks), since bubbles injected into a closed horizontal system do not propagate. In addition, steady-state velocity data could not be obtained in the 34-in. (0.864 m) tanks between 0 and 10 degrees. To elaborate, in traveling along the upper wall, a bubble experiences an outward thrust imparted by the increased pressure in the viscous film between the bubble and the wall. If the magnitude of this thrust, which acts transverse to the wall, is greater than the buoyant force component, the bubble will migrate from the wall. As the bubble travels further from the wall, the thrust diminishes until it equals the buoyant force component. Once this condition is reached, the bubble will propagate at a constant distance from the wall. It was found that this equilibrium position was not reached within the 3-ft (0.914 m) length of the 34-in. (0.864 m) tanks for inclinations between 0 and 10 degrees.

A more detailed description of the experimental investigation is given in the doctoral dissertation of Maneri (1970).

EXPERIMENTAL RESULTS

Rise velocity data were obtained for inclinations ranging from 0 to 85 degrees from the vertical. These data were obtained to investigate bubble rise velocity as a function of tank width, tank spacing, fluid properties, bubble volume, and inclination. Because of the extent of data and possible graphical representations, it was decided that a presentation of the 0, 30 and 80 degree results as curves of velocity versus volume would adequately illustrate most of the above-mentioned effects.

The vertical data are compared with the wave theory correlation of Maneri & Mendelson (1968). To the author's knowledge, this is the only existing correlation which attempts to account for spacing effects. The equation has the form

$$U = U_\infty\left[\tanh\frac{\pi C_0}{2\bar{r}_e + \bar{t}}\right]^{1/2}, \qquad [4]$$

where

$$U_\infty = \left[gb(2\bar{r}_e + \bar{t})\left(1 + \frac{\pi^2}{(2\bar{r}_e + \bar{t})^2\text{Eö}}\right)\right]^{1/2}, \qquad [5]$$

is the rise velocity in a tank having infinite width, and where

$$C_0 = (1 + \bar{t})\tanh^{-1}\left[\frac{\pi\text{F}^2}{(1 + \bar{t})\left(1 + \dfrac{\pi^2}{(1 + \bar{t})^2\text{Eö}}\right)}\right] \qquad [6]$$

is determined from the slug limit which is assumed known *a priori*. Here the dimensionless bubble equivalent radius $\bar{r}_e$, the aspect ratio $\bar{t}$, the Eötvös number Eö and the Froude number F are all based on the tank half-width b. The bubble volume, in terms of $\bar{r}_e$, is given by

$$V = \pi\bar{r}_e^2\bar{t}b^3. \qquad [7]$$

The form of the Maneri–Mendelson equation presented here includes the surface tension effect neglected by the authors in their paper. Furthermore, the slug limit condition has been modified to conform more satisfactorily with experiment. Based on the available experimental data (Zukoski 1966; Nicklin *et al.* 1962), the authors determined C_0 under the assumption that the slug limit was reached when the bubble equivalent radius equalled the tank half-width. The data of the present investigation, however, do not support this assumption but indicate that the slug limit is reached when the equivalent radius is approximately a quarter of the tank width, a result also obtained by Collins (1967) for the cylindrical case.

Finally, the value for the slug Froude number required to determine C_0 is obtained from the analysis of Maneri (1970). In the original expression for C_0, the Froude number was determined from Wallis' (1966) linear correlation, which is given by

$$\text{F} = 0.325 + 0.092\bar{t}, \qquad [8]$$

where the Froude number and the aspect ratio are based on the tank half-width. The analytical results presented by Maneri (1970) show the Froude number is not a simple linear relation of the aspect ratio. Furthermore the analysis shows that surface tension forces cannot be neglected in the spacing direction—a result substantiated by the experimental data of Collins (1965), Griffith (1963) and this investigation. By example, for an aspect ratio of 0.1 and a tank spacing of 0.15 in. (0.381 cm), the predicted Froude number for water at atmospheric conditions would be 15 per cent higher if surface tension is ignored.

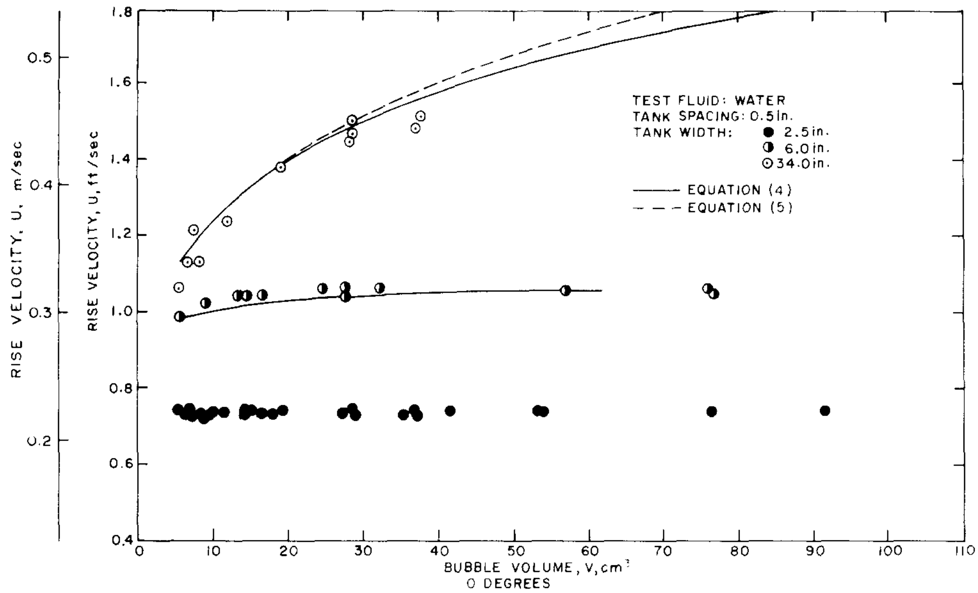
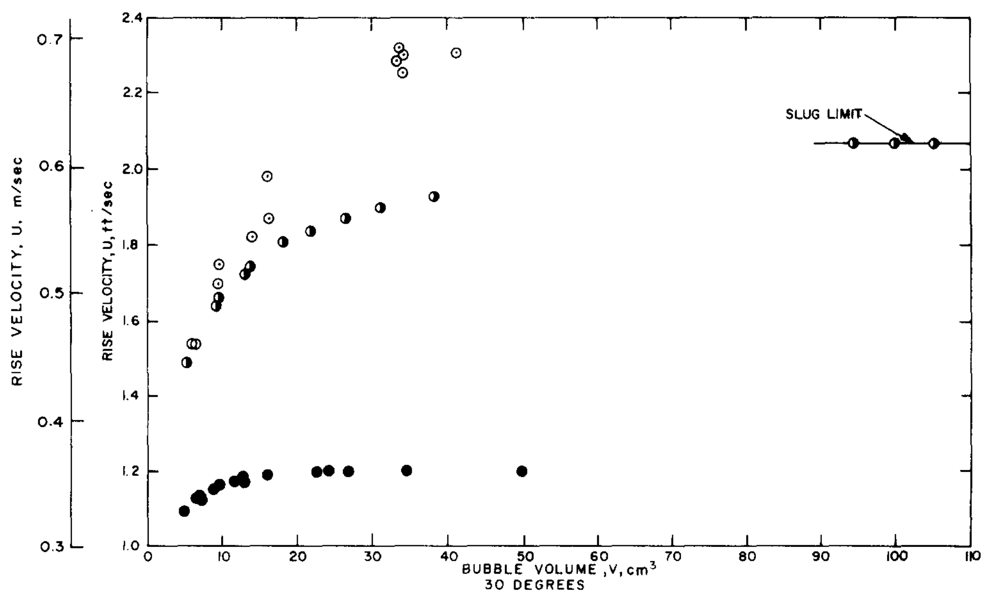


Figure 4. Effect of tank width at inclination.



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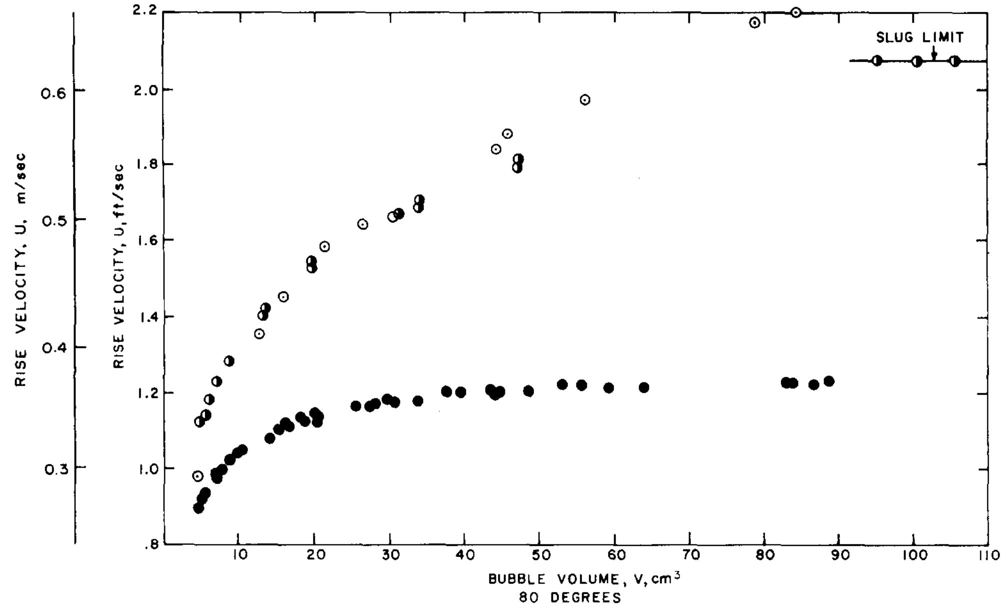

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Effect of tank width

The effect of tank width is given in figure 4 for bubbles rising in water in tanks having a spacing of 0.5 in. (1.27 cm). It is seen that when the tanks are vertical, there is a clear distinction in bubble velocity for each tank width over the entire range of bubble volumes. As the tanks are inclined, the influence of the lower wall of the tank steadily diminishes for a bubble of fixed volume. As seen in figure 4 at 80 degrees inclination, the wall effect has diminished to the point where the velocity in both the 6- and 34-in. (0.152 and 0.864 m) tanks is indistinguishable for volumes below 40 cm^3.

The predictions given by [4] are seen to be in good agreement with the vertical experimental results. The rise velocity of bubbles rising in a tank of infinite width, given by [5], is included for comparison. It shows that the 34-in. (0.864 m) wide tank cannot be considered infinite in extent for bubble volumes greater than about 40 cm^3. The relatively large deviation from the predicted curve at a volume of 104 cm^3 can be attributed to entrance effects since the L/D of this tank was 1.0. In other words, the large shapeless volume of injected air was unable to attain its steady-state shape (hence rise velocity) within the 3-ft (0.914 m) length of the tank.

Effect of tank spacing

The spacing effect is given in figure 5. Disregarding the 34-in. (0.864 m) tank data for the moment, it is seen that a small but measurable spacing effect exists at 0 and 30 degrees inclination, while at 80 degrees inclination there is practically no effect. For the 34-in. (0.864 m) tank no effect is discernible for all three inclinations, a result probably due in part

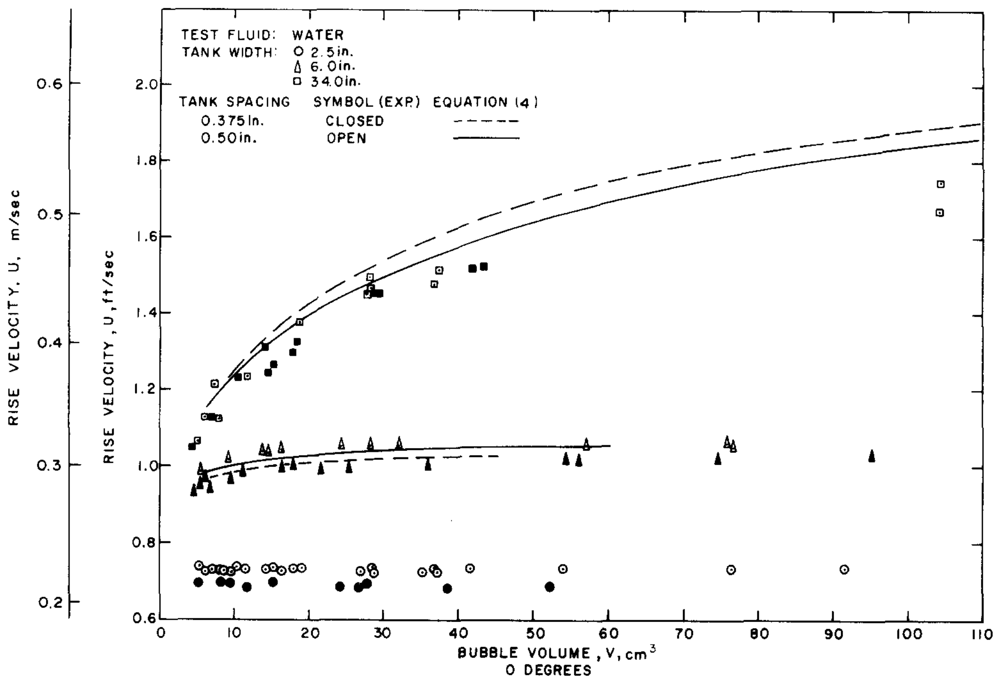
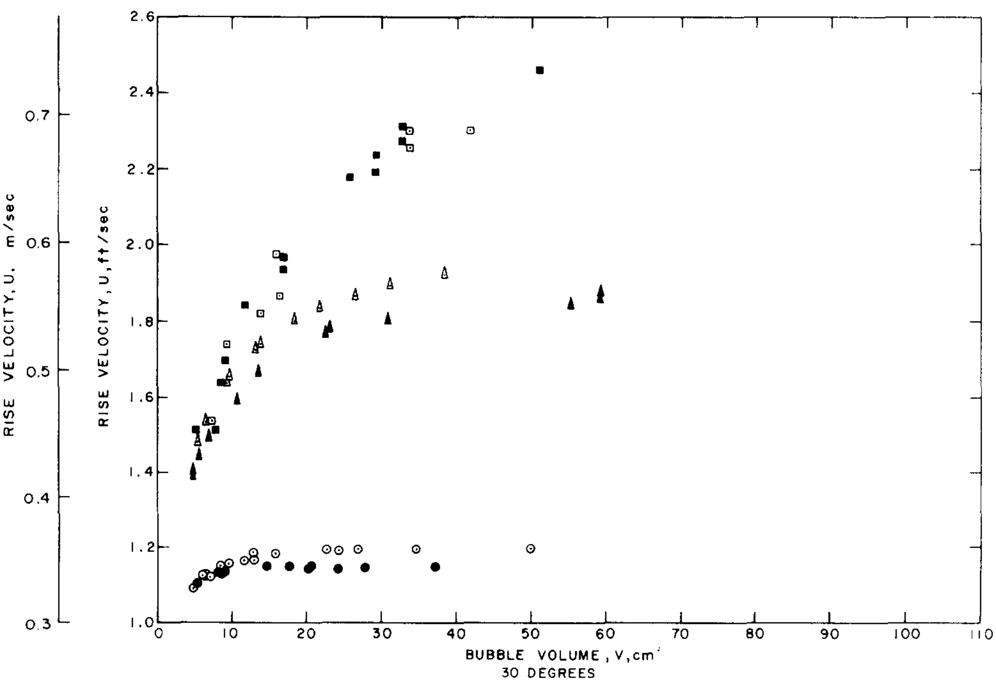


Figure 5. Effect of spacing at inclination for various tank widths.



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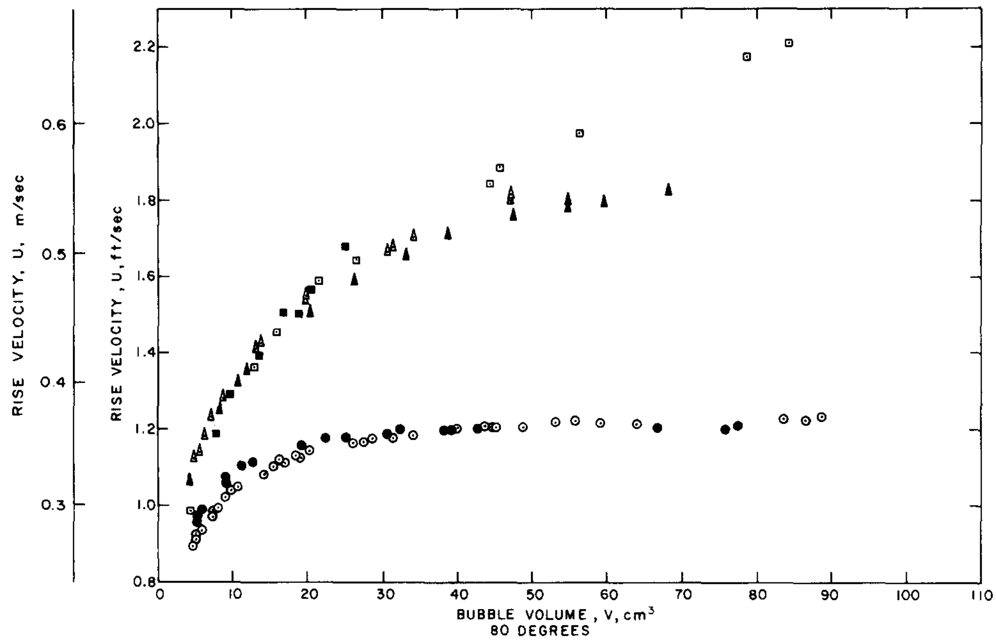


Figure 5. Cont.

to the scatter of the data. This large degree of scatter, relative to the smaller channels, is a result of the previously described difference in the method of volume measurement. The fact that a small effect does exist is inferred by [4], which predicts a decrease in the rise velocity with increasing spacing. This is in contrast to the predictions for the 6-in. (0.152 m) tank which show an increase in velocity with increasing spacing, consistent with the experimental results. Since the entire range of velocities for the 6-in. (0.152 m) tank falls close to the slug limit, the validity of [4] can be questioned for bubble volumes much smaller than the slug limit inception volume.

This seeming discrepancy is resolved when the 34-in. (0.864 m) tank data are compared with [4] on an equivalent radius basis, as given in figure 6. On this basis, it is seen that despite the large degree of scatter, the experimental rise velocity increases with increasing spacing, as is predicted by the wave theory. In view of this, it appears that the equivalent radius allows the rise velocity for two-dimensional tanks to be more consistently correlated.

This contention is substantiated when the 80 degree results of figure 5 are presented on an equivalent radius basis as shown in figure 6. When compared on a volume basis, these results show that for volumes below 30 cm^3, the rise velocity increases with decreasing tank spacing; as the slug limit is approached, the reverse becomes true. However, when compared on an equivalent radius basis the rise velocity either remains the same or increases with increasing spacing. In fact, this behavior is consistently exhibited when the 30 degree data are reduced on the same basis (see figure 6). As a consequence, it seems safe to conclude that on an equivalent radius basis, the rise velocity increases or remains the same with increasing tank spacing, regardless of inclination.

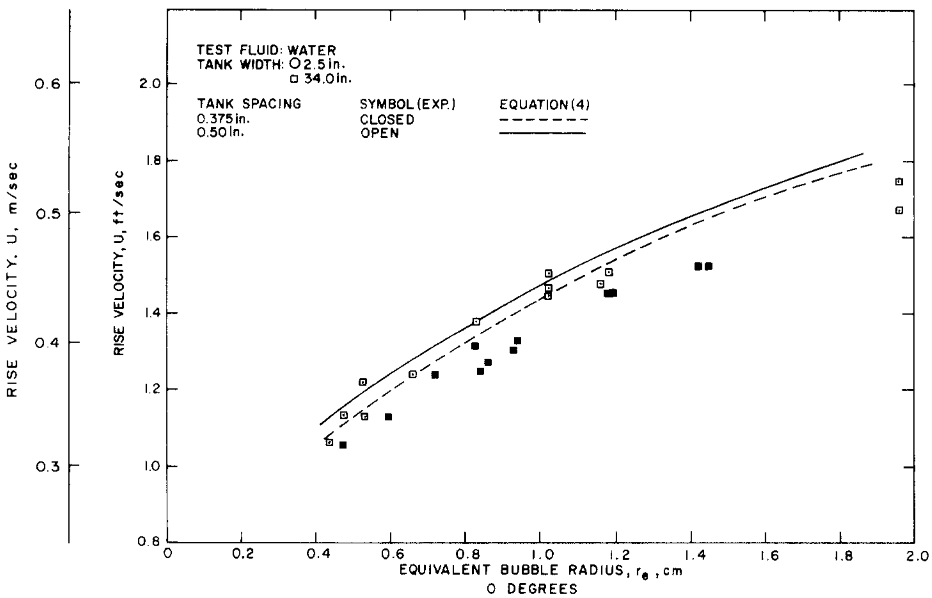
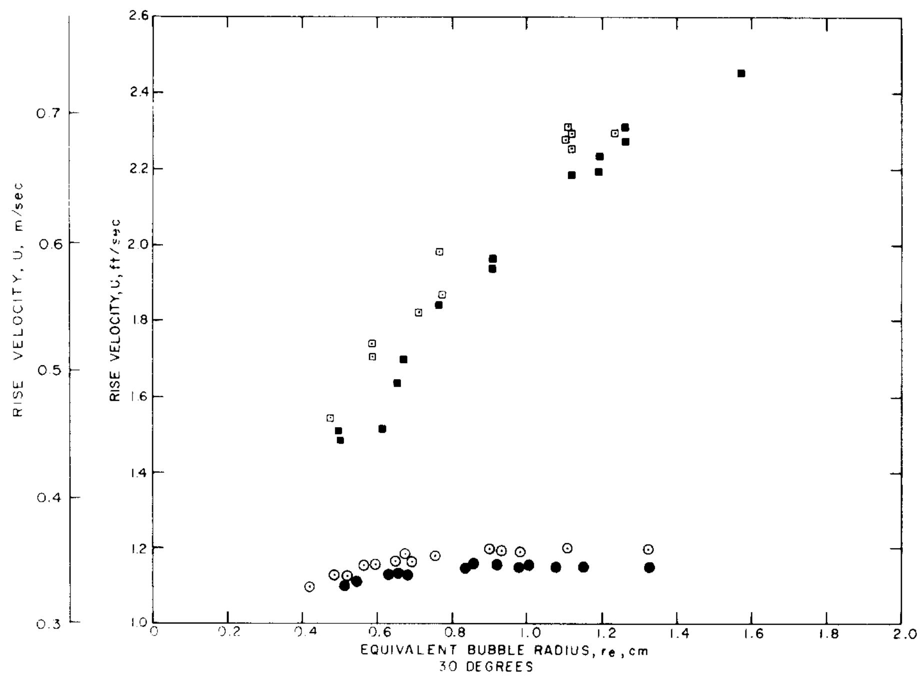


Figure 6. Effect of spacing on an equivalent radius basis.



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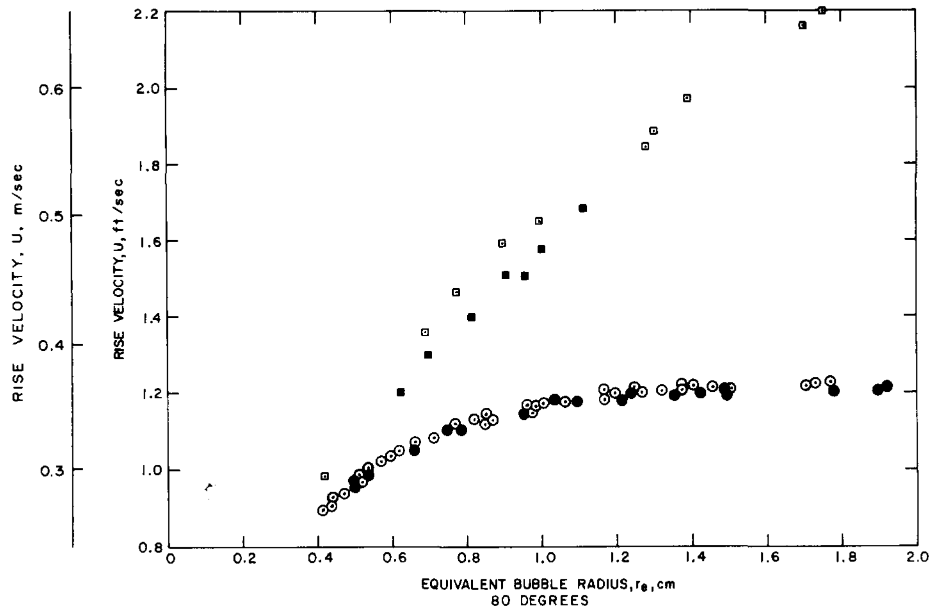

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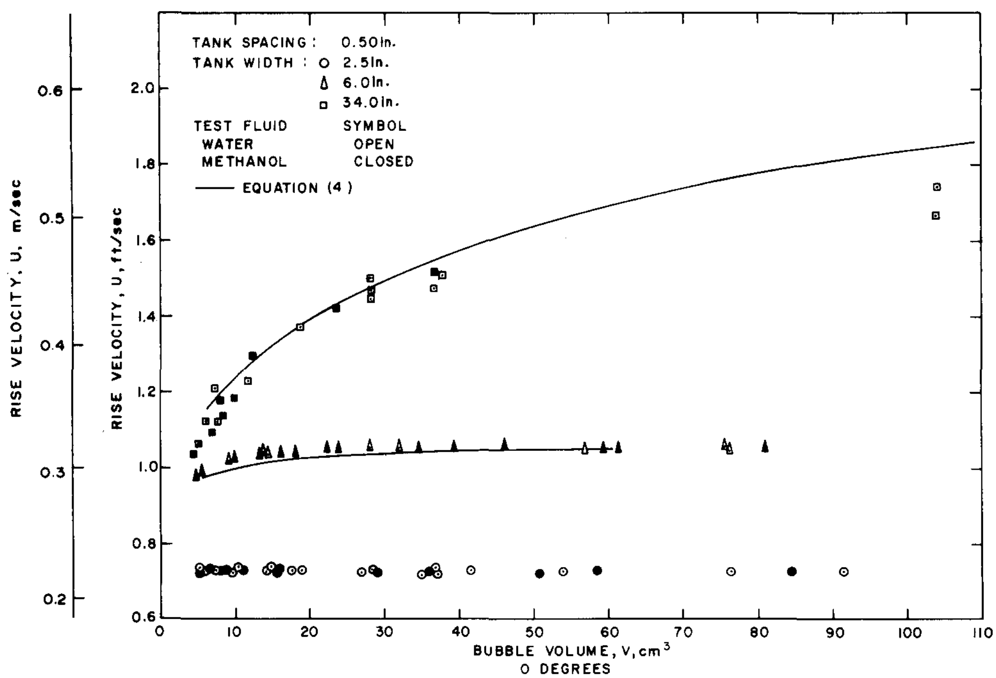
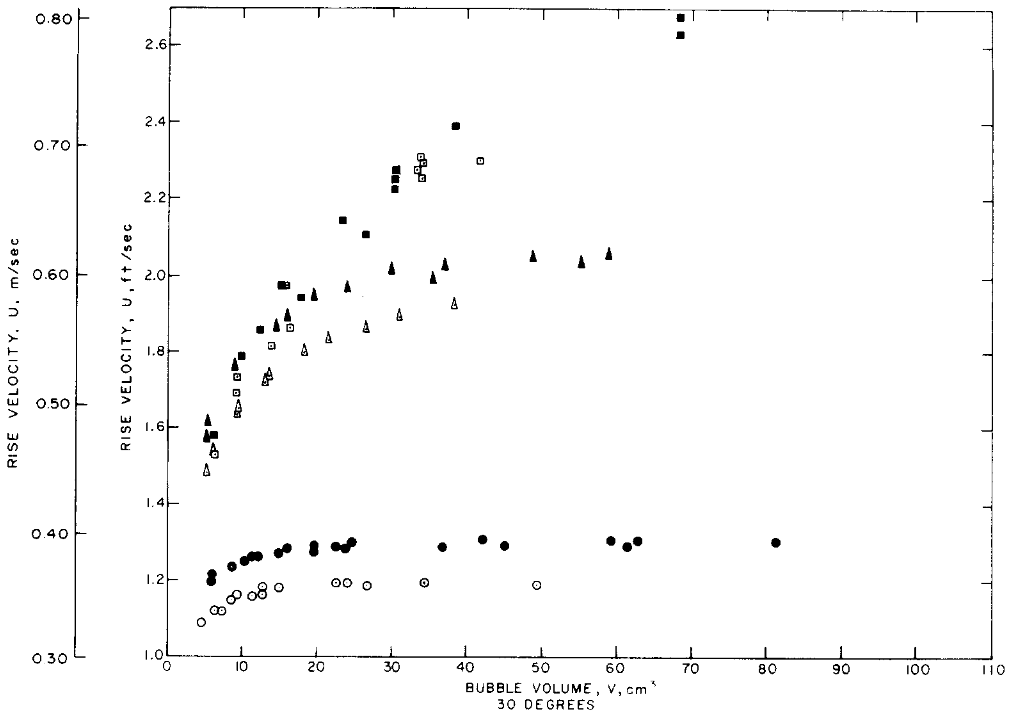


Figure 7. Effect of fluid properties at inclination for various tank widths.



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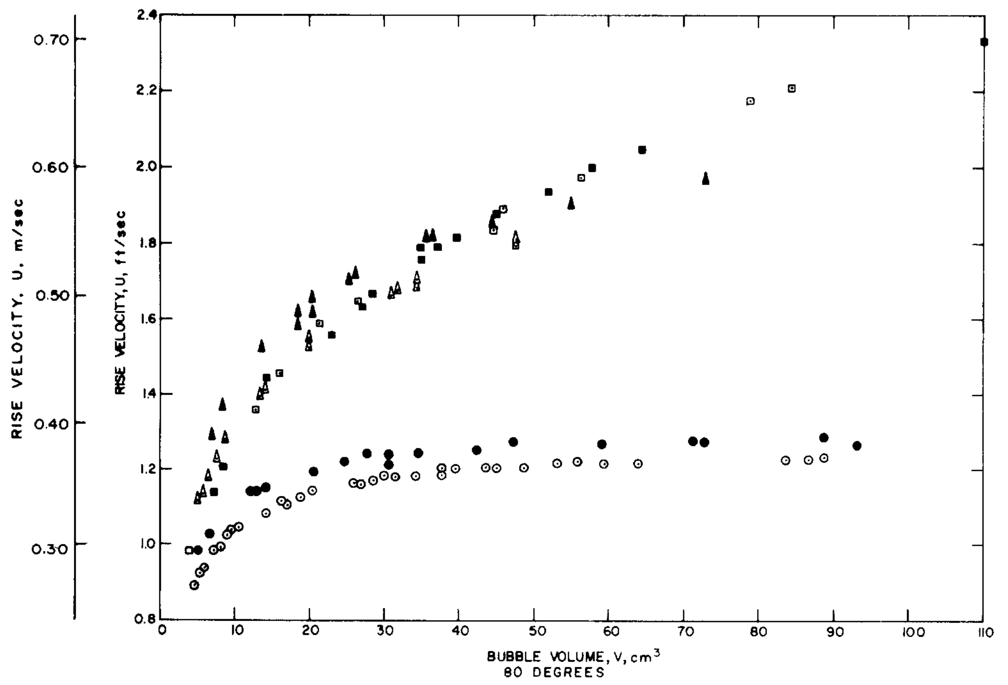


Figure 7. Cont.

Effect of fluid properties

The effect of the difference in properties between water and methanol is demonstrated in figure 7 for inclinations of 0, 30 and 80 degrees. Virtually no difference in the rise velocity is exhibited for all three tanks in the vertical position. At 30 and 80 degrees inclination, however, the methanol results are distinctly higher for the 2.5- and 6-in. (6.35 and 15.24 cm) wide tanks while there is no difference for the 34-in. (0.864 m) wide tank. These observations can be explained in terms of the predominant forces acting on the bubble.

In the vertical case, the predominant forces governing the bubble rise are inertial (for the tank spacing considered); consequently, even though there is a factor of three difference in the surface tensions of the two fluids, no effect is observed. On the other hand, at inclination the large frontal curvature of the bubble in both the spacing and width directions combine to give rise to a substantial surface force. This force is most logically modeled by the Eötvös number (Eö $= \rho g D^2/\sigma$) so that the rise velocity at inclination $U(\theta)$ can be expressed as

$$U(\theta) \sim f(1/\text{Eö}), \qquad [9]$$

where

$$f(1/\text{Eö}) \rightarrow 1\,;\ \text{Eö} \rightarrow \infty. \qquad [10]$$

In view of this, it is evident that for small tank widths, a factor of 3 difference in the surface tension can have a measurable effect, whereas for large tank widths, it is the order of magnitude of the Eötvös number that becomes important. Consequently, a factor of three will not appreciably alter the effect of an Eötvös number of the order of 10^4, as would be the case with the 34-in. (0.864 m) wide tank.

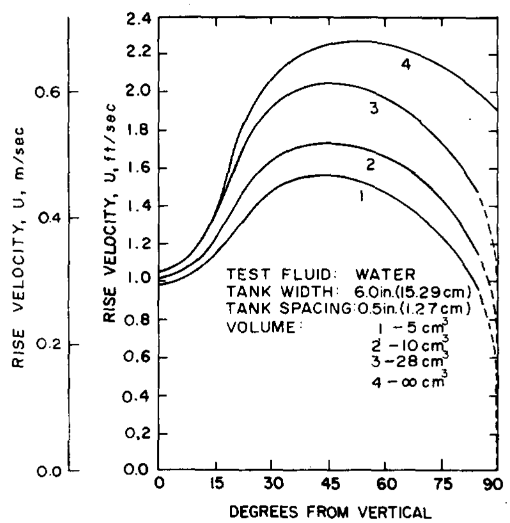


Figure 8. Experimentally observed effect of inclination and bubble volume.

Effect of inclination and bubble volume

The effects of inclination and bubble volume are depicted in figure 8 for water in the 6×0.5-in. (15.24×1.27 cm) tank. The dotted lines between 85 and 90 degrees inclination have been included to indicate that the rise velocity of finite volume bubbles must go to zero at 90 degrees inclination in closed static fluid systems. For inclinations less than or equal to 85 degrees, it is possible to attain the "infinite" bubble velocity limit (slug limit) with finite volume bubbles; however, at 90 degrees inclination, only a bubble of "infinite" extent will propagate at the velocity given in figure 8.

The behavior with inclination can be described qualitatively as the combined result of two competing effects; streamlining of the bubble shape which acts to increase the slug velocity, and a decrease in the axial component of the buoyant force which acts to lower it. Consequently, the Froude number can be expressed as

$$\mathrm{F} = S\sqrt{\cos\theta}, \qquad [11]$$

where S is a shape function expressible in terms of the frontal radius of curvature which in turn is angle dependent.

A good representation of the shape function was obtained experimentally by Grace & Harrison (1967) in their investigation of the effect of frontal radius on vertically rising bubbles. This shape function is qualitatively depicted in figure 9 along with the buoyant force component and the resulting Froude number.

While this representation provides an intuitive description of the inclination angle effect for small bubbles, it is not realistic for bubbles of infinite extent. First, as seen in figure 8, the Froude number of an infinitely long bubble propagating horizontally has a finite value greater than zero. Secondly, the shape function corresponds to the frontal variation of a symmetrical bubble in a nearly potential flow field. Actual bubbles rising at inclination are asymmetrical, with the flow around the bubble becoming progressively one-sided as the

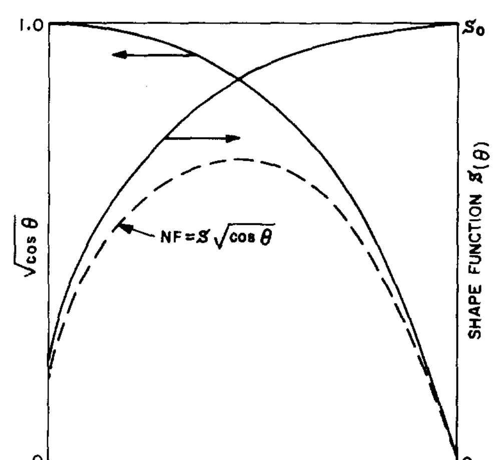


Figure 9. Qualitative representation of inclination variation of Froude number.

inclination is increased. This transfer to flow from a double to a single film suggests interpretation of the velocity-angle behavior from a regime standpoint as shown in figure 10.

The first regime, which extends from 0 to 10 degrees inclination, is characterized by bubbles of relatively large frontal radius. Fluid falls freely around both sides of the bubble, forming two films of constant but different thickness. Only in the special case of vertical flow are the film thicknesses equal. In this first regime, as illustrated in figure 11(a), for aspect ratios (spacing-to-width) less than 0.30 the Froude number increases with increasing aspect ratio; the net result of compensating three-dimensional inertial and surface effects when the spacing is small. A further observation is that, for the same angle, the frontal radius of curvature is effectively a constant fraction of the tank width independent of the tank size and the fluid as seen in table 2.

In the second regime, which extends from 30 to 90 degrees inclination, the transverse component of the buoyant force presses the bubble against the upper edge of the tank resulting in negligible mass flow along the upper surface of the bubble. In contrast to the first regime, the frontal radius of curvature-to-tank width ratio is small, exhibiting both geometrical and fluid properties dependence. The shift from predominantly inertial behavior is further demonstrated in figure 11(b) where it is seen that a distinct dependence of the Froude number on tank width has emerged.

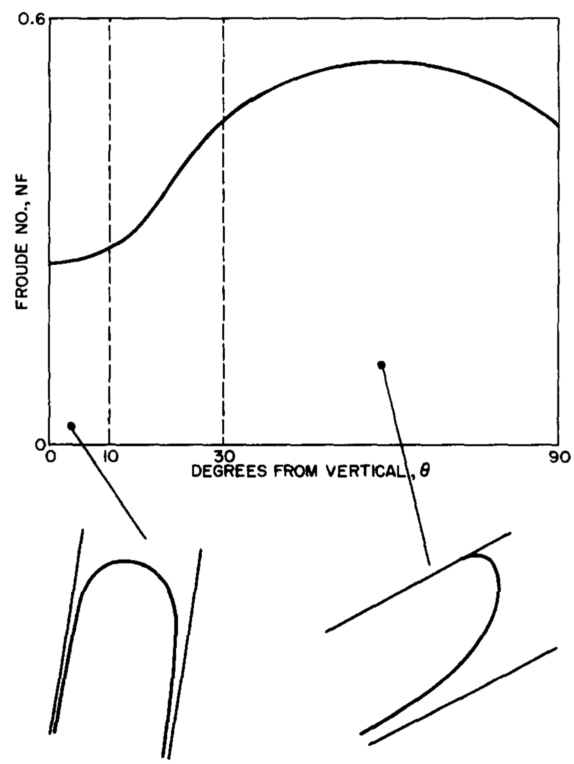


Figure 10. Inclination—dependent bubble flow regimes.

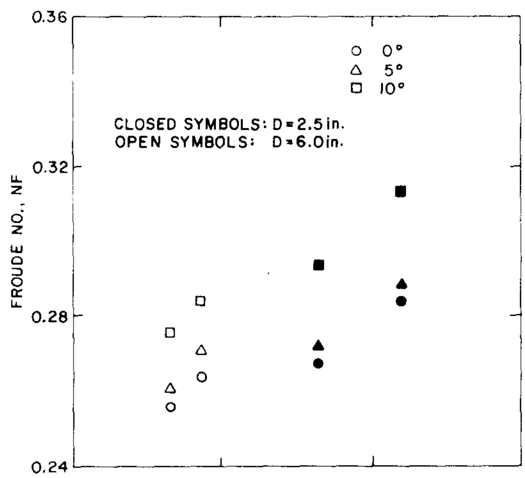
Table 2. Dimensionless frontal radius of curvature a/D—first regime

Width	Spacing	Angle (deg.)	Fluid Water	Fluid Methanol
2.5 in. (6.35 cm)	0.375 in. (0.953 cm)	0	0.318	0.316
		5	0.319	0.332
		10	0.274	0.273
	0.5 in. (1.27 cm)	0	0.325	0.328
		5	—	0.324
		10	0.275	0.278
6.0 in. (15.24 cm)	0.375 in. (0.953 cm)	0	0.313	—
		5	0.314	—
		10	0.285	—
	0.5 in. (1.27 cm)	0	0.323	0.315
		5	0.317	—
		10	0.294	0.291

Spanning the two regimes is a transition region in which the transfer from a blunt nosed bubble-double falling film system to a streamlined bubble-single falling film system is accomplished. The sharp rise in the Froude number is indicative of this transition region.

SUMMARY AND CONCLUSIONS

The characteristics of bubbles rising in two-dimensional tanks have been investigated. Specifically, the dependence of the bubble rise velocity on bubble volume, inclination, tank width, tank spacing, and fluid properties has been experimentally determined for low viscosity fluids.



(a) FIRST REGIME

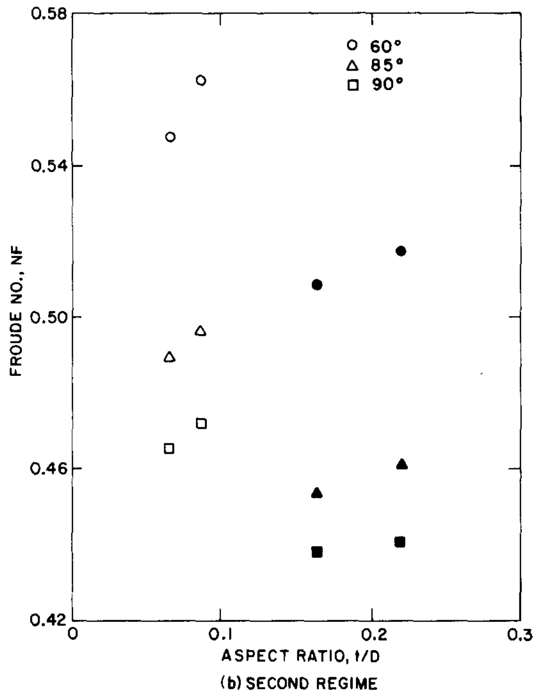


Figure 11. Variation of Froude number with aspect ratio.

The results show that for the vertical case the predominant forces are inertial; however, at inclination, surface and possibly viscous forces become important. In addition, it has been shown experimentally that the three-dimensional character of the flow cannot be ignored.

The experimental results are summarized below:

(1) The edge wall effect decreases as the inclination is increased; thus tanks which exhibit an appreciable wall effect for all bubble sizes when in the vertical position, at inclination have virtually no wall effect for some range of bubble sizes.

(2) The rise velocity of finite volume bubbles is consistently correlated with respect to spacing on an equivalent radius basis, in that the velocity increases or remains the same irrespective of the bubble volume and the inclination.

(3) The rise velocity in fluids of low viscosity is relatively independent of fluid properties in vertical tanks and inclined tanks of infinite extent, whereas in tanks of finite width, the rise velocity at inclination increases with decreasing surface tension.

(4) The motion of inclined bubbles can be interpreted in terms of the three following distinct regimes.

   (a) Inertial dominant—extends from 0 to 10 degrees inclination being characterized by bubbles of relatively large frontal radius whose rise velocity is independent of fluid properties and little changed from the vertical value.

(b) Properties dependent—extends from 30 to 90 degrees inclination being characterized by bubbles whose frontal radius of curvature-to-tank width ratio is relatively small, exhibiting both geometrical and fluid properties dependence and whose rise velocity is approximately twice as large as the vertical value.

(c) Transition—spans the region between 10 and 30 degrees inclination and is characterized by a sharp increase in the rise velocity.

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REFERENCES

BENJAMIN, T. B. 1968 Gravity currents and related phenomena. *J. Fluid Mech.* **31**, 209–247.

COLLINS, R. 1965 A simple model of the plane gas bubble in a finite liquid. *J. Fluid Mech.* **22**, 763–771.

COLLINS, R. 1967 The effect of a containing cylindrical boundary on the velocity of a large gas bubble in a liquid. *J. Fluid Mech.* **28**, 97–112.

DAVIES, R. M. & TAYLOR, G. I. 1950 The mechanics of large bubbles rising through extended liquids and through liquids in tubes. *Proc. R. Soc.* **A200**, 375–390.

DUMITRESCU, D. T. 1943 Stromung an einer Luftblase im senkrechten Rohr. *ZAMM* **23**, 139–149.

GRACE, J. R. & HARRISON, D. 1967 The influence of bubble shape on the rising velocities of large bubbles. *Chem. Engng Sci.* **22**, 1337–1347.

GRIFFITH, P. 1963 The prediction of low-quality boiling voids. A.S.M.E. Paper No. 63-HT-20.

MANERI, C. C. 1970 The motion of plane bubbles in inclined ducts. Ph.D. Thesis, Polytechnic Institute of Brooklyn.

MANERI, C. C. & MENDELSON, H. D. 1968 The rise velocity of bubbles in tubes and rectangular channels as predicted by wave theory. *A.I.Ch.E.J.* **14**, 294–300.

NICKLIN, D. J. *et al.* 1962 Two-phase flow in vertical tubes. *Trans. Instn Chem. Engrs* **40**, 61–68.

RUNGE, D. E. & WALLIS, G. B. 1965 The rise velocity of cylindrical bubbles in inclined tubes. Rpt No. NYO-3114-8.

WALLIS, G. B. & COLLIER, J. B. 1966 Two phase flow and heat transfer. Notes for a summer course July 10–22, Vol. 2, p. 10. Dartmouth College.

WHITE, E. T. & BEARDMORE, R. H. 1962 The velocity of rise of single cylindrical air bubbles through liquids contained in vertical tubes. *Chem. Engng Sci.* **17**, 351–361.

ZUKOSKI, E. E. 1966 Influence of viscosity, surface tension and inclination angle on motion of long bubbles in closed tubes. *J. Fluid Mech.* **25**, 821–837.

Résumé—On étudie les caractéristiques de bulles ascendantes dans un réservoir bi-dimensionnel. On trouve que les propriétés du fluide affectent sensiblement la vitesse des bulles lorsqu' elles s'élèvent obliquement tandis qu'elles ont un effet négligeable sur les bulles lorsqu' elles montent verticalement. L'accroissement de la vitesse d'ascension oblique, observé experimentalement, est expliqué qualitativement. En outre, une comparaison est faite entre la corrélation basée sur l'analogue des vagues de Maneri et Mendelson et des mesures de vitesse d'ascension verticale. Les comparaisons montrent que le corrélation permet de calculer les effets tri-dimensionnels des dimensions du réservoir lorsque les données sont réduites par rapport à un rayon de référence des bulles.

Auszug—Die Charakteristiken von Blasen, die in zweidimensionalen Tanks aufsteigen, werden untersucht. Es wird gefunden, dass die Fluessigkeitseigenschaften, die fuer senkrecht aufsteigender Blasen vernachlaessigbar sind, die Gechwindigkeit geneigt aufsteigender Blasen messbar beeinflussen. Die gemessene Erhoehung der Aufstieggeschwindigkeit unter einer Neigung gegenueber dem wert fuer senkrechten Aufstieg wird qualitativ erklaert. Ferner wird ein Vergleich zwischen der Wellenanalogie-Korrelation von Maneri und Mendelson und Daten der senkrechten Aufstieggeschwindigkeit angestellt. Die Vergleiche zeigen, dass die Korrelation die dreidimensionalen Effekte des Tankabstandes vorhersagt, wenn die Angaben auf der Basis von aequivalentem Blasenradius umgerechnet werden.

Резюме—Исследованы характ еристики пуэырей, подымающихся в двумерных сосудах. Найдено, что влияние свойств жидкости, которое пренебрежимо для вертикально подымающихся пузырей, сказывается на скорости подъема отклоняющихся. Измеренное приращение скорости подъема в зависимости от относительного наклонения по "вертикальному" (нормальному) значению объяснено качественно. В заключение сделано сравнение между волновой аналоговой корреляцией по Манери и Мендельсону и данными вертикальной скорости подъема. Вышесказанное сравнение показывает, что эта корреляция трехмерные явления в полости сосуда при условии уменьшения величин на основе нвивалентного радиуса пузыря.